ANOVA Table showing impact of salary level on IV and DV

		Sum of Squares	Df	Mean Square	F	Sig.
Overall_Motivation	Between Groups	1.496	2	0.748	4.895	0.009
	Within Groups	16.963	111	0.153		
	Total	18.459	113			
Avg_employee performance	Between Groups	0.306	2	0.153	0.553	0.577
	Within Groups	30.692	111	0.277		
	Total	30.998	113			

Independent sample t-test showing impact of gender on IV and DV

Independent Samples Test										
		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Inter	
		F	Sig.	t	df	Sig. (2-tail	Mean Diffe	Std. Error I	Lower	Upper
overall_Motivation	Equal variances assumed	1.758	0.188	0.32	112	0.75	0.02435	0.07612	-0.12646	0.17517
	Equal variances not assumed			0.324	109.28	0.746	0.02435	0.07507	-0.12443	0.17313
Avg_employeeperformance	Equal variances assumed	1.175	0.281	0.24	112	0.811	0.0237	0.09866	-0.17177	0.21918
	Equal variances not assumed			0.242	111.504	0.809	0.0237	0.09776	-0.17001	0.21741

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An Economic Production Quantity (EPQ) Model with Ramp-type Demand and constant Deterioration

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Abstract

In this paper, we construct an Economic Production Quantity model for items with Ramp type demand and constant rate of deterioration. Ramp type demand is most realistic assumption in today's world. Demand is taken as exponential function of time up to certain fixed time. After that demand becomes constant. Production rate is also linear function of demand. Shortages are not allowed. The objective of the study is to minimize the total cost while considering various costs like set-up cost, holding cost and deterioration cost. While minimizing total cost, the optimum production run time is obtained.

Introduction

According to Kimberly Amadeo, “Demand is the consumer's desire and ability to purchase a good or service. Without demand, no business would ever bother producing anything.” When a manufacturing unit thinks of producing a product, number of expenses comes to their consideration. In business, cost is generally monetary estimation of material and resources, efforts and risks incurred, time and utilities consumed, opportunity forgone in manufacturing and delivery of goods and service. These expenses can be categorized as direct costs and indirect costs. Direct costs include money spend on purchasing machinery, purchasing of raw material, salaries to labour, etc. Indirect costs are like rent, interest paid on borrowed money, transportation cost, administrative expenses, deterioration cost, etc. So while planning a business the main objective of a firm is either to reduce cost of production or maximize profit. Pando, V. (2018) et al in their paper discussed inventory model for deteriorating items considering profit maximization. Whereas, Mishra, S. (2012) et al is determined for minimizing the average cost of production per unit time. Chan (2017) et al has also inculcated the concept of

optimal rate of production in their paper for studying a production-inventory model for items deteriorating during delivery.

Different researchers have taken different parameters for studying demand function. Some have discussed the concept of constant demand, others have taken linearly increasing or decreasing demand, time dependent demand, stock dependent demand, exponential demand, etc. Gupta, R. and Vrat, P. (1986), Ray, J. and Chaudhuri, K. (1997), Giri, B. C., Chaudhuri, K. S. (1998**),** Kun-Shan Wu. (2001), Ouyang, L.Y.,(2007) et al considered stock dependent demand. Bose, S., (1995), San-Jose, L.A., (2018) et al have taken time dependent demand. But in actual market conditions it is not so. Demand of a product increases upto a certain level, after that it becomes stagnant. When a new product is launched in the market, with proper advertisement its demand increases but after some time demand stabilizes. Due to this fact there is need of considering more realistic demand function. For this reason, the concept of ramp type demand is used. It means that upto a certain time demand varies as a function of time but after that it becomes constant. Mandal, B., and Pal, A.K., (1998), Skouri, K.,(2009), et al, Wu, K.S. and Ouyang, L.Y., (2000) have discussed inventory model for ramp type demand.

An entrepreneur is a person who develops a business model. He plays a pivotal role in the functioning of the organization. He is considered to be responsible for the success or failure of an organization. Implementation power of important decisions regarding a business lies in his hands. Some of them are: what to produce, for whom to produce and how much to produce. The question how much to produce includes two important parameters: time and quantity. It means it must be decided that upto what time a should continue production. Second parameter includes the optimum quantity to be produced so that neither there are losses due to lost sales nor wastage due to excess inventory storage in godowns. So inventory management is a firms' important responsibility.

In today's competitive world, it is not only sufficient to produce things. The main concern is how to sell the product in the market. Therefore, a number of firms have come forward to help manufacturers sell their product/goods. They are making efforts to make the product available to different sections of society. Nowadays, there is increasing trend of online shopping. In many cities, there are no malls. So it is difficult for customers to buy branded products. But due to the availability of online stores, customers can order all the branded products by staying at home as online stores are providing door to door delivery. All this includes marketing strategies. Due to marketing efforts, with the passage of time demand of a product increases. Sana S.S. and Chaudhuri K.S. (2007), Chowdhury, R.R., (2014) et al, Palanivel, M., Uthayakumar, R. (2015), Shah, N. H. and Vaghela, C. R. (2017), Rathore, H. (2019) has taken advertisement dependent

demand. One other important factor for discussing demand and various costs is Inflation. Inflation can be understood as a fall in purchasing power of currency. Biermann and Thomas (1977) discussed an inventory model taking inflationary conditions.

Mathematical Modelling

In this case, we are considering an economic production quantity model without shortages. Here demand is taken as Ramp type function of time and deterioration is considered to be constant. At initial point, inventory level is zero because there is no production. With the start of production, inventory level goes up in the interval $[0, t_p]$ although there is demand and deterioration. The maximum time for production is taken as $_pt$. At this point, the firm stops manufacturing goods as an optimum amount of inventory is stored. Otherwise, the various costs associated with the production may increase. In the interval $[t_p, T]$, there is no production. Inventory level is depleting due to demand and deterioration and reaches to zero level.

Now, as we are taking ramp type demand, it depends upon time upto a certain level i.e. μ. After that it becomes constant. This μ may occur during production run time or after that. So there arises two cases for μ, (i) $0 \le \mu \le t_p$ (ii) $t_p \le \mu \le T$.

Case (I) When $0 \le \mu \le t_p$, the mathematical equations representing the model are as below:-

$$\frac{dI_1}{dt} + \theta I_1 = (l-1)ae^{bt}, 0 \le t \le \mu, \quad I_1(t) = 0 \tag{1}$$

$$\frac{dI_2}{dt} + \theta I_2 = (l-1)ae^{b\mu}, \ \mu \le t \le t_p \text{ and } I(\mu^-) = I(\mu^+) \tag{2}$$

$$\frac{dI_3}{dt} + \theta I_3 = -ae^{b\mu}, \ t_p \le t \le T \text{ and } I(T) = 0 \tag{3}$$

Solving the above equations and applying the given conditions, we get

$$(1) \Rightarrow I_1(t) = -\frac{(l-1)a}{b+\theta}e^{-\theta t} + \frac{(l-1)a}{b+\theta}e^{bt} \tag{4}$$

$$(2) \Rightarrow I_2(t) = -\frac{(l-1)ab}{\theta(b+\theta)}e^{(b+\theta)\mu-\theta t} - \frac{(l-1)a}{b+\theta}e^{-\theta t} + \frac{(l-1)a}{\theta}e^{b\mu} \tag{5}$$

$$(3) \Rightarrow I_3(t) = \frac{a}{\theta}e^{b\mu+\theta t-\theta t} - \frac{a}{\theta}e^{b\mu} \tag{6}$$

Relation between T and t_p

Using $I(t_p^-) = I(t_p^+)$

$$\Rightarrow -\frac{(l-1)ab}{\theta(b+\theta)}e^{(b+\theta)\mu-\theta t_p} - \frac{(l-1)a}{b+\theta}e^{-\theta t_p} + \frac{(l-1)a}{\theta}e^{b\mu} = \frac{a}{\theta}e^{b\mu+\theta l-\theta t_p} - \frac{a}{\theta}e^{b\mu}$$

$$\Rightarrow -\frac{(l-1)ab}{\theta(b+\theta)}e^{\theta\mu-\theta t_p} - \frac{(l-1)a}{b+\theta}e^{-b\mu-\theta t_p} + \frac{(l-1)a}{\theta} = \frac{a}{\theta}e^{\theta l-\theta t_p} - \frac{a}{\theta}$$

$$\Rightarrow -\frac{(l-1)b}{(b+\theta)}e^{\theta\mu-\theta t_p} - \frac{(l-1)\theta}{b+\theta}e^{-b\mu-\theta t_p} + l - e^{\theta l-\theta t_p} = 0$$

$$\Rightarrow -(l-1)be^{\theta\mu-\theta t_p} - (l-1)\theta e^{-b\mu-\theta t_p} + l(b+\theta) - (b+\theta)e^{\theta l-\theta t_p} = 0$$

$$\Rightarrow -(l-1)b(1+\theta\mu-\theta t_p) - (l-1)\theta(1-b\mu-\theta t_p) + l(b+\theta) - (b+\theta)(1+\theta T-\theta t_p) = 0$$

$$\Rightarrow T = lt_p \qquad (7)$$

Holding cost, HC in time interval $[0,T]$ is calculated as below

$$HC = h\int_0^T e^{-\delta t}I(t)dt = h\left\{\int_0^{\mu} e^{-\delta t}I_1(t)dt + \int_{\mu}^{t_p} e^{-\delta t}I_2(t)dt + \int_{t_p}^{T} e^{-\delta t}I_3(t)dt\right\}$$

$$= h\left\{\int_0^{\mu}\left[-\frac{(l-1)a}{b+\theta}e^{-(\delta+\theta)t} + \frac{(l-1)a}{b+\theta}e^{(b-\delta)t}\right]dt + \int_{\mu}^{t_p}\left[-\frac{(l-1)ab}{\theta(b+\theta)}e^{(b+\theta)\mu-(\theta+\delta)t} - \frac{(l-1)a}{b+\theta}e^{-(\theta+\delta)t} + \frac{(l-1)a}{\theta}e^{b\mu-\delta t}\right]dt + \right.$$
$$\left.\int_{t_p}^{T}\left[\frac{a}{\theta}e^{b\mu+\theta l-(\theta+\delta)t} - \frac{a}{\theta}e^{b\mu-\delta t}\right]dt\right\}$$

$$= h\left\{\left[\frac{(l-1)a}{(b+\theta)(\delta+\theta)}e^{-(\delta+\theta)t} + \frac{(l-1)a}{(b+\theta)(b-\delta)}e^{(b-\delta)t}\right]_0^{\mu} + \right.$$
$$\left.\left[\frac{(l-1)ab}{\theta(b+\theta)(\theta+\delta)}e^{(b+\theta)\mu-(\theta+\delta)t} + \frac{(l-1)a}{(b+\theta)(\theta+\delta)}e^{-(\theta+\delta)t} - \frac{(l-1)a}{\delta\theta}e^{b\mu-\delta t}\right]_{\mu}^{t_p} + \left[-\frac{a}{\theta(\theta+\delta}e^{b\mu+\theta l-(\theta+\delta)t} + \frac{a}{\theta\delta}e^{b\mu-\delta t}\right]_{t_p}^{T}\right\}$$

Using $T = lt_p$ in above integration and simplifying, we get

$$= h\left\{\begin{array}{l}\frac{(l-1)a}{(b+\theta)}e^{(b-\delta)\mu}\left(\frac{1}{b-\delta} - \frac{b}{\theta(\theta+\delta)} + \frac{b+\theta}{\theta\delta}\right) - \frac{(l-1)a}{(b-\delta)(\delta+\theta)} + \frac{(l-1)ab}{\theta(b+\theta)(\theta+\delta)}e^{(b+\theta)\mu-(\theta+\delta)lt_p} + \frac{(l-1)a}{(b+\theta)(\theta+\delta)}e^{-(\theta+\delta)lt_p} \\ -\frac{al}{\delta\theta}e^{b\mu-\delta lt_p} + \frac{a}{\theta(\theta+\delta)}e^{b\mu+\theta lt_p-(\theta+\delta)lt_p} + \frac{a}{(\theta+\delta)\delta}e^{b\mu-\delta lt_p}\end{array}\right\}$$

Deterioration cost in time interval $[0,T]$ is as below

$$DC = \theta c_p\int_0^T e^{-\delta t}I(t)dt$$

$$= \theta c_p\left\{\begin{array}{l}\frac{(l-1)a}{(b+\theta)}e^{(b-\delta)\mu}\left(\frac{1}{b-\delta} - \frac{b}{\theta(\theta+\delta)} + \frac{b+\theta}{\theta\delta}\right) - \frac{(l-1)a}{(b-\delta)(\delta+\theta)} + \frac{(l-1)ab}{\theta(b+\theta)(\theta+\delta)}e^{(b+\theta)\mu-(\theta+\delta)lt_p} + \frac{(l-1)a}{(b+\theta)(\theta+\delta)}e^{-(\theta+\delta)lt_p} \\ -\frac{al}{\delta\theta}e^{b\mu-\delta lt_p} + \frac{a}{\theta(\theta+\delta)}e^{b\mu+\theta lt_p-(\theta+\delta)lt_p} + \frac{a}{(\theta+\delta)\delta}e^{b\mu-\delta lt_p}\end{array}\right\}$$

The total inventory cost in time interval $[0,T]$ is therefore given by

$$TIC(t_p) = K + HC + DC$$

$$\Rightarrow TIC(t_p) = K + (h+\theta c_p)\left\{\begin{array}{l}\frac{(l-1)a}{(b+\theta)}e^{(b-\delta)\mu}\left(\frac{1}{b-\delta} - \frac{b}{\theta(\theta+\delta)} + \frac{b+\theta}{\theta\delta}\right) - \frac{(l-1)a}{(b-\delta)(\delta+\theta)} + \\ \frac{(l-1)ab}{\theta(b+\theta)(\theta+\delta)}e^{(b+\theta)\mu-(\theta+\delta)lt_p} + \frac{(l-1)a}{(b+\theta)(\theta+\delta)}e^{-(\theta+\delta)lt_p} \\ -\frac{al}{\delta\theta}e^{b\mu-\delta lt_p} + \frac{a}{\theta(\theta+\delta)}e^{b\mu+\theta lt_p-(\theta+\delta)lt_p} + \frac{a}{(\theta+\delta)\delta}e^{b\mu-\delta lt_p}\end{array}\right\}$$

To find the extremum of Total Inventory Cost $TIC(t_p)$, differentiate $TIC(t_p)$, w.r.t. (t_p), we get

$$\frac{d(TIC(t_p))}{dt_p} = (h+\theta c_p)\left\{\begin{array}{l}-\frac{(l-1)ab}{\theta(b+\theta)}e^{(b+\theta)\mu-(\theta+\delta)lt_p} - \frac{(l-1)a}{(b+\theta)}e^{-(\theta+\delta)lt_p} + \frac{al}{\theta}e^{b\mu-\delta lt_p} + \frac{a}{\theta(\theta+\delta)}(\theta l-(\theta+\delta))e^{b\mu+\theta lt_p-(\theta+\delta)lt_p} \\ -\frac{al}{\theta+\delta}e^{b\mu-\delta lt_p}\end{array}\right\}$$

and $\frac{d(TIC(t_p))}{dt_p} = 0$

$$\Rightarrow [l(l-1)(\theta-\delta)-\theta l^2]t_p^2 + 2lt_p - 2b\mu^2 = 0$$

Finding the second derivative of TAC w.r.t. (t_p), we get

$$\frac{d^2(TIC)}{dt_p^2}=(h+\theta c_p)\left\{\begin{array}{l}\frac{(l-1)ab(\theta+\delta)}{\theta(b+\theta)}e^{(b+\theta)\mu-(\theta+\delta)t_p}+\frac{(l-1)a(\theta+\delta)}{(b+\theta)}e^{-(\theta+\delta)t_p}-\frac{la\delta}{\theta}e^{b\mu-\delta t_p}+\\ \frac{a}{\theta(\theta+\delta)}(\theta l-(\theta+\delta))^2e^{b\mu+\theta lt_p-(\theta+\delta)t_p})+\frac{a}{\theta+\delta}(\delta l^2e^{b\mu-\delta lt_p})\end{array}\right\}$$

As all the terms in second derivative are positive, therefore by the principle of maxima and minima, value of $TAC(t_p)$ is minimum.

Case (II) When $t_p \le \mu \le T$, the mathematical equations representing the model are as below:-

$$\frac{dI_1}{dt}+\theta I_1=(l-1)ae^{bt},\ 0\le t\le t_p,\ \ I(t)=0 \qquad (8)$$

$$\frac{dI_2}{dt}+\theta I_2=-ae^{bt},\ t_p\le t\le \mu \text{ and } I(t_p^-)=I(t_p^+) \qquad (9)$$

$$\frac{dI_3}{dt}+\theta I_3=-ae^{b\mu},\ \mu\le t\le T \text{ and } I(T)=0 \qquad (10)$$

Solving the above equations and applying the given conditions, we get

$$(8)\Rightarrow I_1(t)=-\frac{(l-1)a}{b+\theta}e^{-\theta t}+\frac{(l-1)a}{b+\theta}e^{bt} \qquad (11)$$

$$(9)\Rightarrow I_2(t)=\frac{la}{(b+\theta)}e^{(b+\theta)t_p-\theta t}-\frac{(l-1)a}{b+\theta}e^{-\theta t}-\frac{a}{b+\theta}e^{bt} \qquad (12)$$

$$(10)\Rightarrow I_3(t)=\frac{a}{\theta}e^{b\mu+\theta T-\theta t}-\frac{a}{\theta}e^{b\mu} \qquad (13)$$

Relation between T and t_p

Using $I(\mu^-)=I(\mu^+)$

$$\Rightarrow \frac{la}{(b+\theta)}e^{(b+\theta)t_p-\theta\mu}-\frac{(l-1)a}{b+\theta}e^{-\theta\mu}-\frac{a}{b+\theta}e^{b\mu}=\frac{a}{\theta}e^{b\mu+\theta T-\theta\mu}-\frac{a}{\theta}e^{b\mu}$$

$$\Rightarrow \frac{la}{(b+\theta)}(1+(b+\theta)t_p-\theta\mu)-\frac{(l-1)a}{b+\theta}(1-\theta\mu)-\frac{a}{b+\theta}(1+b\mu)=\frac{a}{\theta}(1+b\mu+\theta T-\theta\mu)-\frac{a}{\theta}(1+b\mu)$$

$$\Rightarrow T=lt_p \qquad (14)$$

Holding cost, HC and Deterioration cost, DC in time interval $[0,T]$ is calculated as below

$$=(h+\theta c_p)\left\{\begin{array}{l}\int_0^{t_p}\left[-\frac{(l-1)a}{b+\theta}e^{-(\theta+\delta)t}+\frac{(l-1)a}{b+\theta}e^{(b-\delta)t}\right]dt+\\ \int_{t_p}^{\mu}\left[\frac{la}{(b+\theta)}e^{(b+\theta)t_p-(\theta+\delta)t}-\frac{(l-1)a}{b+\theta}e^{-(\theta+\delta)t}-\frac{a}{b+\theta}e^{(b-\delta)t}\right]dt+\\ \int_{\mu}^{T}\left[\frac{a}{\theta}e^{b\mu+\theta T-(\theta+\delta)t}-\frac{a}{\theta}e^{b\mu-\delta t}\right]dt\end{array}\right\}$$

Using $T=lt_p$ in above integration and simplifying, we get

$$HC+DC=(h+\theta c_p)\left\{\begin{array}{l}\frac{al}{(\delta+\theta)(b-\delta)}e^{(b-\delta)lt_p}-\frac{(l-1)a}{(b-\delta)(\theta+\delta)}-\frac{la}{(b+\theta)(\theta+\delta)}e^{(b+\theta)t_p-(\theta+\delta)\mu}+\\ \frac{(l-1)a}{(b+\theta)(\theta+\delta)}e^{-(\theta+\delta)\mu}-ae^{(b-\delta)\mu}\left(\frac{1}{(b+\theta)(b-\delta)}+\frac{1}{\theta\delta}\right)\\ +\frac{a}{\theta(\theta+\delta)}e^{b\mu+\theta lt_p-(\theta+\delta)\mu}+\frac{a}{\delta(\theta+\delta)}e^{b\mu-\delta lt_p}\end{array}\right\}$$

The total inventory cost in time interval $[0,T]$ is therefore given by

$$TIC(t_p)=K+HC+DC$$

$$\Rightarrow$$

$$TIC(t_p)=K+(h+\theta c_p)\left\{\begin{array}{l}\frac{al}{(\delta+\theta)(b-\delta)}e^{(b-\delta)lt_p}-\frac{(l-1)a}{(b-\delta)(\theta+\delta)}-\frac{la}{(b+\theta)(\theta+\delta)}e^{(b+\theta)t_p-(\theta+\delta)\mu}+\\ \frac{(l-1)a}{(b+\theta)(\theta+\delta)}e^{-(\theta+\delta)\mu}-ae^{(b-\delta)\mu}\left(\frac{1}{(b+\theta)(b-\delta)}+\frac{1}{\theta\delta}\right)\\ +\frac{a}{\theta(\theta+\delta)}e^{b\mu+\theta lt_p-(\theta+\delta)\mu}+\frac{a}{\delta(\theta+\delta)}e^{b\mu-\delta lt_p}\end{array}\right\}$$

To find the extremum of Total Inventory Cost $TIC(t_p)$, differentiate $TIC(t_p)$ w.r.t. (t_p), we get

$$\frac{d(TIC)}{dt_p} = 0$$

$$\Rightarrow e^{(h-\delta)t_p} - e^{(h+\theta)t_p-(\theta+\delta)\mu} - e^{h\mu-\delta(t_p} + e^{h\mu+\theta t_p-(\theta+\delta)\mu} = 0$$

$$\frac{d(TIC)}{dt_p} = (h+\theta c_p)\frac{al}{\theta+\delta}\left\{e^{(h-\delta)t_p} - e^{(h+\theta)t_p-(\theta+\delta)\mu} - e^{h\mu-\delta(t_p} + e^{h\mu+\theta t_p-(\theta+\delta)\mu}\right\}$$

Finding the second derivative of TIC w.r.t. (t_p), we get

$$\frac{d^2(TIC)}{dt_p^2} = (h+\theta c_p)\frac{al}{\theta+\delta}\left\{(b-\delta)e^{(h-\delta)t_p} - (b+\theta)e^{(h+\theta)t_p-(\theta+\delta)\mu} + \delta l e^{h\mu-\delta(t_p} + \theta l e^{h\mu+\theta t_p-(\theta+\delta)\mu}\right\}$$

As all the terms in second derivative are positive, therefore by the principle of maxima and minima, value of $TIC(t_p)$ is minimum.

EPQ Model with Ramp type demand and marketing efforts

Upto this level, we have discussed EPQ model with ramp type demand and constant deterioration. In the next model, we will add marketing efforts to demand function. Here marketing efforts are again considered to be ramp type function of time. The new demand function becomes

$$D(t) = \begin{cases} ae^{bt} + \alpha + \beta t, & t < \mu \\ ae^{b\mu} + \alpha + \beta\mu, & t \geq \mu \end{cases}$$

This function represents that demand and marketing efforts increase with the increase in time but upto a certain level i.e. μ. After that demand becomes stable and with this a company also stabilizes its marketing/promotional efforts. Here again, the objective of this work is to find the value of μ for which the company has optimum production level keeping the Total Cost minimum. So, there arises two possibilities for μ, (i) $0 \leq \mu \leq t_p$ (ii) $t_p \leq \mu \leq T$.

Sub-Case (I) When $0 \leq \mu \leq t_p$, the mathematical equations representing the model are as below:-

$$\frac{dI_1}{dt} + \theta I_1 = (l-1)(ae^{bt} + \alpha + \beta t), 0 \leq t \leq \mu, \quad I(0) = 0 \qquad (15)$$

$$\frac{dI_2}{dt} + \theta I_2 = (l-1)(ae^{b\mu} + \alpha + \beta\mu), \ \mu \leq t \leq t_p \text{ and } I(\mu^-) = I(\mu^+) \qquad (16)$$

$$\frac{dI_3}{dt} + \theta I_3 = -(ae^{b\mu} + \alpha + \beta\mu), \ t_p \leq t \leq T \text{ and } I(T) = 0 \qquad (17)$$

Solving the above equations and applying the given conditions, we get

$$(15) \Rightarrow I_1(t) = \frac{(l-1)a}{b+\theta}(e^{bt} - e^{-\theta t}) + (l-1)\left(\frac{\alpha+\beta t}{\theta}\right) + \frac{(l-1)\beta}{\theta^2}(e^{-\theta t} - 1) - \frac{(l-1)\alpha e^{-\theta t}}{\theta} \qquad (18)$$

$(16) \Rightarrow$

$$I_2(t) = (l-1)e^{-\theta t}\left[-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu} - \frac{a}{b+\theta} + \frac{\beta}{\theta^2}(1-e^{\theta\mu}) - \frac{\alpha}{\theta}\right] + \frac{(l-1)(ae^{b\mu} + \alpha + \beta\mu)}{\theta} \qquad (19)$$

$$(17) \Rightarrow I_3(t) = \left(\frac{ae^{b\mu} + \alpha + \beta\mu}{\theta}\right)(e^{\theta T - \theta t} - 1) \qquad (20)$$

Relation between T and t_p

Using $I(t_p^-) = I(t_p^+)$

$$\Rightarrow (l-1)e^{-\theta t_p}\left[-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu} - \frac{a}{b+\theta} + \frac{\beta}{\theta^2}(1-e^{\theta\mu}) - \frac{\alpha}{\theta}\right] + \frac{(l-1)(ae^{b\mu} + \alpha + \beta\mu)}{\theta} =$$

$$\left(\frac{ae^{b\mu} + \alpha + \beta\mu}{\theta}\right)(e^{\theta l - \theta t_p} - 1)$$

$$\Rightarrow (l-1)\left[-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu} - \frac{a}{b+\theta} + \frac{\beta}{\theta^2}(1-e^{\theta\mu}) - \frac{\alpha}{\theta}\right] = \frac{(ae^{b\mu} + \alpha + \beta\mu)}{\theta}(e^{\theta T} - le^{\theta t_p})$$

$$\Rightarrow (l-1)\left[-\frac{ab}{\theta(b+\theta)}(1+(b+\theta)\mu) - \frac{a}{b+\theta} + \frac{\beta}{\theta^2}(-\theta\mu) - \frac{\alpha}{\theta}\right] = \frac{(ae^{b\mu} + \alpha + \beta\mu)}{\theta}(1+\theta T - l - l\theta t_p)$$

$$\Rightarrow T = lt_p \qquad (21)$$

Holding cost, HC and deterioration cost, DC in time interval $[0,T]$ is calculated as below

$$\text{HC+DC} = (h+\theta c_p)\int_0^T e^{-\delta t} I(t)dt = (h+\theta c_p)\left\{\int_0^{\mu} e^{-\delta t} I_1(t)dt + \int_{\mu}^{t_p} e^{-\delta t} I_2(t)dt + \int_{t_p}^{T} e^{-\delta t} I_3(t)dt\right\}$$

Solving the above equations and applying the given conditions, we get

$$(22)\Rightarrow I_1(t)=-(l-1)\left(\frac{a}{b+\theta}+\frac{\alpha}{\theta}-\frac{\beta}{\theta^2}\right)e^{-\theta t}+(l-1)\left(\frac{ae^{bt}}{b+\theta}+\frac{1}{\theta}\left(\alpha+\beta t-\frac{\beta}{\theta}\right)\right) \quad (25)$$

$$(23)\Rightarrow$$

$$I_2(t)=-(l-1)\left(\frac{a}{b+\theta}+\frac{\alpha}{\theta}-\frac{\beta}{\theta^2}\right)e^{-\theta t}+\frac{a}{b+\theta}\left(le^{(b+\theta)t_p-\theta t}-e^{bt}\right)+\frac{1}{\theta}\left(\alpha+\beta t-\frac{\beta}{\theta}\right)\left(le^{\theta(t_p-t)}-1\right)$$

$$(24)\Rightarrow I_3(t)=\left(\frac{ae^{b\mu}+\alpha+\beta\mu}{\theta}\right)\left(e^{\theta t-\theta t}-1\right) \quad (26)$$

Relation between T and t_p

Using $I(\mu^-)=I(\mu^+)$

$$\Rightarrow lt_p\left(a+\alpha+\beta\mu-\frac{\beta}{\theta}\right)=(ae^{b\mu}+\alpha+\beta\mu)T-ab\mu^2-\frac{l\beta\mu}{\theta} \quad (27)$$

Holding cost, HC and deterioration cost, DC in time interval $[0,T]$ is calculated as below

$$\text{HC+DC}=\left(h+\theta c_p\right)\left\{\int_0^{t_p}e^{-\delta t}I(t)dt+\int_{t_p}^{\mu}e^{-\delta t}I(t)dt+\int_{\mu}^{T}e^{-\delta t}I(t)dt\right\}$$

$$=\left(h+\theta c_p\right)\left\{\begin{array}{l}\left[\frac{ale^{(b-\delta)t_p}}{(\delta+\theta)(b-\delta)}-\frac{l}{(\theta+\delta)\delta}\left(\alpha+\beta t_p-\frac{\beta}{\theta}+\frac{\beta}{\delta}+\frac{\beta}{\theta+\delta}\right)e^{-\delta t_p}-\frac{(l-1)}{(\theta+\delta)}\left(\frac{a}{b-\delta}-\frac{\alpha}{\delta}-\frac{\beta}{\delta^2}\right)\right]+\\ \frac{(l-1)}{\theta+\delta}\left(\frac{a}{(b+\theta)}+\frac{\alpha}{\theta}-\frac{\beta}{\theta^2}\right)e^{-(\theta+\delta)\mu}-\frac{al}{(b+\theta)(\delta+\theta)}e^{(b+\theta)t_p-(\theta+\delta)\mu}-\frac{a}{(b+\theta)(b-\delta)}e^{(b-\delta)\mu}\\ -\frac{l}{\theta(\theta+\delta)}e^{\theta t_p-(\theta+\delta)\mu}\left(\alpha+\beta\mu-\frac{\beta}{\theta}+\frac{\beta}{\theta+\delta}\right)+\left(\frac{\beta}{\theta\delta^2}-\frac{\beta}{\theta^2\delta}-\frac{ae^{b\mu}}{\theta\delta}\right)e^{-\delta\mu}+\\ \frac{(ae^{b\mu}+\alpha+\beta\mu)}{\theta}\left(\frac{e^{\theta T-(\theta+\delta)\mu}}{\theta+\delta}+\frac{\theta}{(\theta+\delta)\delta}e^{-\delta T}\right)\end{array}\right\}$$

The total inventory cost in time interval $[0,T]$ is therefore given by

$$TIC(t_p)=K+HC+DC$$

$$\Rightarrow$$

$$TIC(t_p)=K+\left(h+\theta c_p\right)\left\{\begin{array}{l}\left[\frac{ale^{(b-\delta)t_p}}{(\delta+\theta)(b-\delta)}-\frac{l}{(\theta+\delta)\delta}\left(\alpha+\beta t_p-\frac{\beta}{\theta}+\frac{\beta}{\delta}+\frac{\beta}{\theta+\delta}\right)e^{-\delta t_p}-\frac{(l-1)}{(\theta+\delta)}\left(\frac{a}{b-\delta}-\frac{\alpha}{\delta}-\frac{\beta}{\delta^2}\right)\right]+\\ \frac{(l-1)}{\theta+\delta}\left(\frac{a}{(b+\theta)}+\frac{\alpha}{\theta}-\frac{\beta}{\theta^2}\right)e^{-(\theta+\delta)\mu}-\frac{al}{(b+\theta)(\delta+\theta)}e^{(b+\theta)t_p-(\theta+\delta)\mu}-\frac{a}{(b+\theta)(b-\delta)}e^{(b-\delta)\mu}\\ -\frac{l}{\theta(\theta+\delta)}e^{\theta t_p-(\theta+\delta)\mu}\left(\alpha+\beta\mu-\frac{\beta}{\theta}+\frac{\beta}{\theta+\delta}\right)+\left(\frac{\beta}{\theta\delta^2}-\frac{\beta}{\theta^2\delta}-\frac{ae^{b\mu}}{\theta\delta}\right)e^{-\delta\mu}+\\ \frac{(ae^{b\mu}+\alpha+\beta\mu)}{\theta}\left(\frac{e^{\theta T-(\theta+\delta)\mu}}{\theta+\delta}+\frac{\theta}{(\theta+\delta)\delta}e^{-\delta T}\right)\end{array}\right\}$$

To find the extremum of Total Inventory Cost $TIC(t_p)$, differentiate $TIC(t_p)$ w.r.t. (t_p), we get

$$\frac{d(TIC)}{dt_p}=0$$

$$\Rightarrow-\beta\delta t_p^{\,2}+\left[-(\theta+\delta)(a+\alpha)+\beta+\frac{\beta\delta}{\theta}-\beta\mu\theta+\frac{l\left(a+\alpha+\beta\mu-\frac{\beta}{\theta}\right)^2(\theta+\delta)}{(ae^{b\mu}+\alpha+\beta\mu)}\right]t_p \quad \text{[Using (27)]}$$

$$+\frac{\left(a+\alpha+\beta\mu-\frac{\beta}{\theta}\right)(\theta+\delta)}{(ae^{b\mu}+\alpha+\beta\mu)}\left(ab\mu^2+\frac{l\beta\mu}{\theta}\right)=0$$

Appendix A

$$\frac{d^2(TIC)}{dt_p^{\,2}}=\left(h+\theta c_p\right)\frac{al}{\theta}\left\{be^{bt_p}-(b+\theta)e^{(b+\theta)t_p-\theta\mu}+\theta le^{b\mu+\theta t_p-\theta\mu}\right\}$$

$$=\left(h+\theta c_p\right)\frac{al}{\theta}\left\{be^{bt_p}-(b+\theta)e^{bt_p}\left(1+\theta t_p-\theta\mu\right)+\theta le^{b\mu+\theta t_p-\theta\mu}\right\}$$

$$=\left(h+\theta c_p\right)al\left\{e^{bt_p}\left[(\mu-t_p)(b+\theta)-1\right]+le^{b\mu+\theta t_p-\theta\mu}\right\},$$

which is positive as this case is for $(t_p\le\mu\le T)$. It implies $\mu-t_p\ge 0$

$$= (h+\theta c_p)\left\{\begin{aligned}&\int_0^{\mu}\left[\frac{(l-1)\alpha}{b+\theta}(e^{(b-\delta)t}-e^{-(\theta+\delta)t})+(l-1)\left(\frac{\alpha+\beta t}{\theta}\right)e^{-\delta t}+\frac{(l-1)\beta}{\theta^2}(e^{-(\theta+\delta)t}-e^{-\delta t})-\frac{(l-1)\alpha e^{-(\theta+\delta)t}}{\theta}\right]dt+\\&\int_{\mu}^{t_p}\left[(l-1)e^{-(\theta+\delta)t}\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+\frac{(l-1)(ae^{b\mu}+\alpha+\beta\mu)e^{-\delta t}}{\theta}\right]dt+\\&\int_{t_p}^{T}\left[\left(\frac{ae^{b\mu}+\alpha+\beta\mu}{\theta}\right)(e^{\theta t-(\theta+\delta)t}-e^{-\delta t})\right]dt\end{aligned}\right.$$

$$= (h+\theta c_p)\left\{\begin{aligned}&\left[\frac{(l-1)\alpha}{b+\theta}\left(\frac{e^{(b-\delta)t}}{b-\delta}+\frac{e^{-(\theta+\delta)t}}{\theta+\delta}\right)-(l-1)\left(\frac{\alpha+\beta t}{\theta\delta}\right)e^{-\delta t}+\frac{(l-1)\beta}{\theta^2}\left(-\frac{e^{-(\theta+\delta)t}}{\theta+\delta}+\frac{e^{-\delta t}}{\delta}\right)+\frac{(l-1)\alpha e^{-(\theta+\delta)t}}{\theta(\theta+\delta)}\right]_0^{\mu}\\&\left[-(l-1)\frac{e^{-(\theta+\delta)t}}{\theta+\delta}\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)-\frac{(l-1)(ae^{b\mu}+\alpha+\beta\mu)e^{-\delta t}}{\theta\delta}\right]_{\mu}^{t_p}\\&+\left[\left(\frac{ae^{b\mu}+\alpha+\beta\mu}{\theta}\right)\left(-\frac{e^{\theta t-(\theta+\delta)t}}{\theta+\delta}+\frac{e^{-\delta t}}{\delta}\right)\right]_{t_p}^{T}\end{aligned}\right.$$

Using $T = lt_p$ in above integration and simplifying, we get

$$= (h+\theta c_p)\left\{\begin{aligned}&\left[\frac{(l-1)\alpha}{b+\theta}e^{(b-\delta)\mu}\left(\frac{1}{b-\delta}-\frac{1}{(\theta+\delta)\theta}\right)+\frac{(l-1)}{(\theta+\delta)}\left(\frac{a}{b-\delta}+\frac{\beta}{\delta^2}+\frac{\alpha}{\delta}\right)-\frac{(l-1)\beta}{\delta^2(\theta+\delta)}e^{-\delta\mu}\right]+\\&\left[-(l-1)\frac{e^{-(\theta+\delta)t_p}}{\theta+\delta}\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+\frac{(l-1)\alpha}{\theta\delta}e^{b\mu-\delta\mu}\right]\\&+\left[(ae^{b\mu}+\alpha+\beta\mu)\left(\frac{e^{-\delta lt_p}}{(\theta+\delta)\delta}-\frac{le^{-\delta t_p}}{\theta\delta}+\frac{e^{\theta lt_p-(\theta+\delta)t_p}}{(\theta+\delta)\theta}\right)\right]\end{aligned}\right\}$$

The total inventory cost in time interval $[0,T]$ is therefore given by

$$TIC(t_p) = K + HC + DC$$

$\Rightarrow$

$$TIC(t_p) = K + (h+\theta c_p)\left\{\begin{aligned}&\left[\frac{(l-1)\alpha}{b+\theta}e^{(b-\delta)\mu}\left(\frac{1}{b-\delta}-\frac{1}{(\theta+\delta)\theta}\right)+\frac{(l-1)}{(\theta+\delta)}\left(\frac{a}{b-\delta}+\frac{\beta}{\delta^2}+\frac{\alpha}{\delta}\right)-\frac{(l-1)\beta}{\delta^2(\theta+\delta)}e^{-\delta\mu}\right]+\\&\left[-(l-1)\frac{e^{-(\theta+\delta)t_p}}{\theta+\delta}\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+\frac{(l-1)\alpha}{\theta\delta}e^{b\mu-\delta\mu}\right]\\&+\left[(ae^{b\mu}+\alpha+\beta\mu)\left(\frac{e^{-\delta lt_p}}{(\theta+\delta)\delta}-\frac{le^{-\delta t_p}}{\theta\delta}+\frac{e^{\theta lt_p-(\theta+\delta)t_p}}{(\theta+\delta)\theta}\right)\right]\end{aligned}\right\}$$

To find the extremum of Total Inventory Cost $TIC(t_p)$, differentiate $TIC(t_p)$ w.r.t. (t_p), we get

$$\frac{d(TIC)}{dt_p} = (h+\theta c_p)\left\{\begin{aligned}&(l-1)e^{-(\theta+\delta)t_p}\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+\\&(ae^{b\mu}+\alpha+\beta\mu)\left[\frac{-le^{-\delta lt_p}}{(\theta+\delta)}+\frac{le^{-\delta t_p}}{\theta}+\frac{e^{\theta lt_p-(\theta+\delta)t_p}(\theta l-(\theta+\delta))}{(\theta+\delta)\theta}\right]\end{aligned}\right\}$$

$$\frac{d(TIC)}{dt_p} = 0$$

$$\Rightarrow t_p^2\{(l-1)\frac{(\theta+\delta)^2}{2}\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+(ae^{b\mu}+\alpha+\beta\mu)\left(\frac{-\delta^2l^3}{2(\theta+\delta)}+\frac{\delta^2l}{2\theta}\right)$$

$$+\left(\frac{l}{\theta+\delta}-\frac{1}{\theta}\right)\left(\frac{\theta l-(\theta+\delta)^2}{2}\right)\}+t_p\{-(l-1)(\theta+\delta)\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+$$

$$(ae^{b\mu}+\alpha+\beta\mu)\left(\frac{\delta l^2}{(\theta+\delta)}-\frac{\delta l}{\theta}\right)+\left(\frac{l}{\theta+\delta}-\frac{1}{\theta}\right)(\theta l-(\theta+\delta))\}+$$

$$\{(l-1)\left(-\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+(ae^{b\mu}+\alpha+\beta\mu)\left(\frac{-l}{(\theta+\delta)}+\frac{l}{\theta}\right)+\left(\frac{l}{\theta+\delta}-\frac{1}{\theta}\right)\}=0$$

Finding the second derivative of TAC w.r.t. (t_p), we get

$$\frac{d^2(TIC)}{dt_p^2} = (h+\theta c_p)\left\{\begin{aligned}&(l-1)(\theta+\delta)e^{-(\theta+\delta)t_p}\left(\frac{ab}{\theta(b+\theta)}e^{(b+\theta)\mu}+\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(e^{\theta\mu}-1)+\frac{\alpha}{\theta}\right)+\\&(ae^{b\mu}+\alpha+\beta\mu)\left[\frac{\delta le^{-\delta t_p}}{\theta}\left(\frac{l\theta e^{-(l-1)\delta t_p}}{(\theta+\delta)}-1\right)+\frac{e^{\theta lt_p-(\theta+\delta)t_p}(\theta l-(\theta+\delta))^2}{(\theta+\delta)\theta}\right]\end{aligned}\right\}$$

Sub-Case (II) When $t_p \le \mu$, the mathematical equations representing the model are as below:-

$$\frac{dI_1}{dt}+\theta I_1 = (l-1)(ae^{bt}+\alpha+\beta t),\ 0\le t\le t_p,\ I(0)=0 \qquad (22)$$

$$\frac{dI_2}{dt}+\theta I_2 = -(ae^{bt}+\alpha+\beta t),\ t_p\le t\le\mu \text{ and } I(t_p^-)=I(t_p^+) \qquad (23)$$

$$\frac{dI_3}{dt}+\theta I_3 = -(ae^{b\mu}+\alpha+\beta\mu),\ \mu\le t\le T \text{ and } I(T)=0 \qquad (24)$$

$$e^{-\theta t_p}\left\{(I-1)e^{-\theta t_p}\left(-\frac{ab}{\theta(b+\theta)}e^{(t-\theta)\mu}-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}(1-e^{\theta\mu})-\frac{\alpha}{\theta}\right)+\right.$$
$$\left.(ae^{b\mu}+\alpha+\beta\mu)\left[\frac{-Ie^{\theta t_p-\theta t_p}}{(\theta+\delta)}+\frac{I}{\theta}+\frac{e^{\theta t_p-\theta t_p}(\theta I-(\theta+\delta))}{(\theta+\delta)\theta}\right]\right\}=0$$

$$\Rightarrow(I-1)(1-\theta t_p+\frac{\theta^2 t_p^{\ 2}}{2!})\left(-\frac{ab}{\theta(b+\theta)}(1+(b+\theta)\mu+\frac{(b+\theta)^2\mu^2}{2!})-\frac{a}{b+\theta}+\frac{\beta}{\theta^2}\left(1-1-\theta\mu-\frac{\theta^2\mu^2}{2!}\right)-\frac{\alpha}{\theta}\right)+$$
$$(a(1+b\mu+\frac{b^2\mu^2}{2!})+\alpha+\beta\mu)\left[\frac{-I(1-\theta t_p(I-1)+\frac{1}{2!}\delta^2 t_p^{\ 2}(I-1)^2}{(\theta+\delta)}+\right.$$
$$\left.\frac{I}{\theta}+\left(1+\theta t_p(I-1)+\frac{\theta^2 t_p^{\ 2}(I-1)^2}{2!}\right)\left(\frac{I}{\theta+\delta}-\frac{1}{\theta}\right)\right]=0$$

Conclusion

In this model, we have found the optimal time schedule at which firm will stop production so that minimum cost is incurred. Optimum production run time for the situation when efforts by sales team included are also incorporated It is due to the fact that with increase in efforts by sales team, there is increase in demand which leads to requirement of maintaining higher inventory level. The nature of varied situations in different combinations affects the management standpoint on production lot size measure.

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